

Name: _____ Date: _____ Class: _____

ESSENTIAL QUESTION

How do slope and intercepts let us write, graph, and interpret linear equations — and how do we use real data to build and evaluate a linear model?

Unit Overview

Unit 2 focused on linear relationships — the simplest and most widely used mathematical model. Slope-intercept form ($y=mx+b$) and point-slope form ($y-y_1=m(x-x_1)$) let us write the equation of any line from a slope and a point, or from two points. Intercepts show where a line crosses each axis. In real-world contexts, slope is a rate of change and the y-intercept is a starting value. When data isn't perfectly linear, we use a scatter plot and a line of best fit (linear regression) to model the trend and make predictions.

$$\begin{aligned} y &= mx + b \text{ (slope-intercept)} \\ y - y_1 &= m(x - x_1) \text{ (point-slope)} \\ m &= (y_2 - y_1)/(x_2 - x_1) \end{aligned}$$

Slope	Rate of change — rise over run; undefined for vertical lines, 0 for horizontal
Model	In context: slope = rate of change, y-intercept = starting/fixed value
Best Fit	Line minimizing the sum of squared residuals (vertical distances to points)

Quick Reference

- x-intercept: set $y=0$ and solve for x . y-intercept: set $x=0$ and solve for y .
- To write an equation from two points: find slope first, then use point-slope form with either point.
- Interpolation (predicting within the data range) is more reliable than extrapolation (predicting far outside it).

Worked Examples**EXAMPLE 1**

Write the equation of the line through (2,5) and (6,13) in slope-intercept form.

- Find slope: $m = (13-5)/(6-2) = 8/4 = 2$
- Point-slope with (2,5): $y - 5 = 2(x-2)$
- Simplify: $y = 2x - 4 + 5$

Answer: $y = 2x + 1$

EXAMPLE 2

Find the x- and y-intercepts of $3x + 4y = 12$.

- x-intercept: set $y=0 \rightarrow 3x=12 \rightarrow x=4$
- y-intercept: set $x=0 \rightarrow 4y=12 \rightarrow y=3$

Answer: x-intercept: (4,0); y-intercept: (0,3)

EXAMPLE 3

A company's cost is a \$50 startup fee plus \$12 per item made. Write a model $C(x)$ and find the cost for 15 items.

- Fixed cost 50 is the y-intercept; rate \$12/item is the slope
- $C(x) = 12x + 50$
- $C(15) = 12(15) + 50 = 180 + 50$

Answer: $C(x) = 12x + 50$; $C(15) = \$230$

EXAMPLE 4

A car's value is $V(t) = 25000 - 1800t$. Interpret the slope and intercept, and find when $V=0$.

- Slope -1800: the car's value decreases \$1800 per year
- y-intercept 25000: the car's initial value
- Set $V=0$: $0 = 25000 - 1800t \rightarrow t = 25000/1800$

Answer: Value reaches \$0 after $t \approx 13.9$ years

EXAMPLE 5

Data: (1,12) (2,15) (3,19) (4,22) (5,26). Find the line of best fit and predict y at $x=8$.

- Mean $x=3$, mean $y=18.8$; slope $= \Sigma(x-\bar{x})(y-\bar{y}) / \Sigma(x-\bar{x})^2 = 35/10 = 3.5$
- Intercept $= 18.8 - 3.5(3) = 8.3 \rightarrow y \approx 3.5x + 8.3$
- Predict at $x=8$: $y = 3.5(8) + 8.3 = 28 + 8.3$

Answer: $y \approx 3.5x + 8.3$; prediction at $x=8$ is $y \approx 36.3$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

- 1** Write the equation of the line through (1,4) and (5,-8) in slope-intercept form.

Hint: Find the slope first, then use point-slope form with either point.

- 2 Find the x- and y-intercepts of $2x - 5y = 20$.

Hint: Set $y=0$ to find the x-intercept; set $x=0$ to find the y-intercept.

- 3 A gym charges a \$30 sign-up fee plus \$25/month. Write a model $M(x)$ and find the cost after 8 months.

Hint: The fixed fee is the y-intercept; the monthly rate is the slope.

- 4 A temperature model is $T(h) = 70 - 3h$. Interpret the slope and intercept, then find when $T=40$.

Hint: Slope is the rate of change per hour; intercept is the starting temperature. Set $T=40$ and solve for h .

- 5 Data: (0,5) (2,11) (4,17) (6,23). Verify the data is exactly linear and find the equation.

Hint: Check that the slope between each consecutive pair of points is the same.

Key Vocabulary

Slope	The rate of change of a line: rise over run, $m = (y_2 - y_1)/(x_2 - x_1)$.
Slope-Intercept Form	A linear equation written as $y = mx + b$, where m is slope and b is the y-intercept.
Point-Slope Form	A linear equation written as $y - y_1 = m(x - x_1)$, using a known point and slope.
Intercept	The point where a line crosses an axis — set the other variable to 0 to find it.
Linear Model	A real-world relationship represented by a linear equation, with slope as rate and intercept as starting value.
Scatter Plot	A graph of individual data points used to visualize the relationship between two variables.
Line of Best Fit	The line that minimizes the total squared vertical distance to a set of data points.
Extrapolation	Using a model to predict values outside the range of the original data — less reliable than interpolation.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 Find the slope of the line through (2,3) and (5,12).

Multiple Choice

A 3**B** $\frac{1}{3}$ **C** 9**D** -3**2 Write the equation of the line with slope -2 through (1,4) in slope-intercept form.**

Multiple Choice

A $y = -2x + 6$ **B** $y = -2x + 2$ **C** $y = 2x + 6$ **D** $y = -2x - 6$ **3 Find the x-intercept of $4x + 3y = 24$.**

Multiple Choice

A 6**B** 8**C** 24**D** 3**4 A linear model is $P(t) = 1500 + 45t$, where t is months. What does the slope represent?**

Multiple Choice

A The initial value of 1500**B** P increases by 45 each month**C** P decreases by 45 each month**D** The starting month is 45**5 What does a line of best fit minimize?**

Multiple Choice

A The x -values**B** The sum of squared residuals**C** The slope**D** The y -intercept

Common Mistakes to Avoid

✗ Mixing up slope-intercept and point-slope form, or substituting the wrong values.

✓ In point-slope form, (x_1, y_1) is a specific point on the line — substitute it directly, no extra solving needed first.

✗ Jumping straight to $y = mx + b$ when only two points are given, without finding slope first.

✓ Always compute $m = (y_2 - y_1) / (x_2 - x_1)$ first, then use either point to find b .

✗ Mixing up the x -intercept and y -intercept, or forgetting to set the OTHER variable to 0.

✓ x -intercept: set $y = 0$ and solve for x .
 y -intercept: set $x = 0$ and solve for y .

✗ Using a linear model to predict far outside the range of the original data with full confidence.

✓ Predictions inside the data range (interpolation) are more reliable than those far outside it (extrapolation).

Math Tips

- ★ Slope = rise/run = $\Delta y / \Delta x$. Keep the subtraction order consistent in the numerator and denominator.
- ★ Point-slope form is fastest when you have a point and a slope, or two points, without solving for b right away.
- ★ To graph a line quickly: plot the y -intercept, then use the slope to find a second point.
- ★ In a real-world model, the y -intercept is often the 'starting value' and the slope is the 'rate of change.'
- ★ A line of best fit doesn't have to pass through any actual data point — it minimizes overall error.